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A Translational Theorem for the Class of EOL Languages

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If K is not a context-free language, then $\mathrm{sh}(K, a^*)$ is not an EOL language (where $\mathrm{sh}(K_1, K_2)$ denotes the shuffle of the languages K_1 and K_2, and a is a symbol not in the alphabet of K). Hence the class of context-free languages is the largest full AFL inside the class of EOL languages.

1. INTRODUCTION

Classes of languages generated by L systems without nonterminals [9] are not closed with respect to many operations considered in formal language theory. However, for L systems with nonterminals closure under the usual AFL operations can often be shown. Thus, e.g., both the class of ETOL languages and the class of ETOL languages of finite index are full AFLs. On the other hand, the class of EOL languages, one of the main L-families, is not a full AFL although it is "almost one": it is closed under union, concatenation, star, and gsm mappings, but not under arbitrary finite-state transductions. In other words, EOL is closed under intersection with regular sets and under finite substitution, but neither under regular substitution nor under inverse homomorphisms.

This nonclosure under inverse homomorphisms was first proved in [5] where it is shown that the language $\mathrm{sh}(K_0, a^*)$ with $K_0 = \{b^{2^n} \mid n \geqslant 0\}$ is not in EOL, where $\mathrm{sh}(K, a^*)$ is the result of shuffling an arbitrary number of a's (a not appearing in K) into the strings of K. Note that $K_0 \in \mathrm{EOL}$, and that the operation $\mathrm{sh}(-, a^*)$ is both a regular substitution and an inverse homomorphism. The proof in [5] is strongly oriented towards this particular language.

Then in [2] a combinatorial property of EOL languages was proved, which as a corollary yields that $\mathrm{sh}(K, a^*)$ is not an EOL language whenever K is "numerically dispersed", which roughly means that the length-set of K is of exponential nature. Clearly a language K satisfying this assumption is not context-free.

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In this paper we extend this result by demonstrating that $\mathrm{sh}(K, a^*)$ is not an EOL language whenever K is not context-free. Thus, intuitively speaking, it is difficult to combine the insertion of "rubbish" (i.e., all the a's) with the kind of parallel rewriting that is used in EOL systems. An immediate consequence of this result is that CF, the class of context-free languages, is the largest full AFL inside EOL.

Since our result is of the form "if $\mathrm{sh}(K, a^*) \in \mathrm{EOL}$ then $K \in \mathrm{CF}$", it is a translational theorem (or a bridge-theorem), often used in language theory to prove proper inclusions between classes of languages. The syntactic lemma from [4] is a well-known example of such a theorem. In [10] a translational theorem for EOL languages is proved concerning the operation of copying. In [3, 7] it is shown that $\mathrm{sh}(K, a^*)$ is not in EDTOL whenever K is not an EDTOL language of finite index.

The paper is divided into four sections, this introduction being the first. In Section 2 we define EOL systems (in a nonstandard way) and state a lemma which expresses the well-known technique of slicing an EOL system. In Section 3 we prove the main result and mention some of its consequences. Finally, in Section 4, we slightly generalize the main result and show that, in this new form, it implies the above-mentioned combinatorial property of EOL languages proved in [2].

2. Definitions and a Lemma

For a string w, $|w|$ denotes its length. For an alphabet T, we denote by pres_T any homomorphism that preserves T, i.e., such that $\mathrm{pres}_T(t) = t$ for all $t \in T$ and $\mathrm{pres}_T(a) = \lambda$ for all $a \notin T$ (where λ is the empty string). For a language $K \subseteq T^*$ and a symbol $a \notin T$, the result of shuffling a's into K is defined by $\mathrm{sh}(K, a^*) = \mathrm{pres}_T^{-1}(K) = \{w \in (T \cup \{a\})^* \mid \mathrm{pres}_T(w) \in K\}$. Note that $\mathrm{sh}(K, a^*) = \mathrm{rub}(K)$, where rub is the regular substitution such that $\mathrm{rub}(t) = a^* t a^*$ for all $t \in T$ (to be precise, a^* has to be added to $\mathrm{rub}(K)$ in case $\lambda \in K$).

We assume the reader to be familiar with the basic theory of context-free grammars [6] and EOL systems [9]. CF denotes the class of context-free languages and EOL denotes the class of EOL languages.

The main result of this paper provides a bridge between EOL and CF. For this reason we define an EOL system in such a way that it is closely related to a context-free grammar.

An *EOL system* is a construct $G = (N, T, R, S)$, where N is a finite set of nonterminals, T is a finite set of terminals ($N \cap T = \varnothing$), $S \in N$ is the initial nonterminal, and R is a finite set of rules of the form $A \to \psi$, where $A \in N$ and $\psi \in (N - \{S\})^* \cup T^*$. Thus the rules can be divided into nonterminal and terminal rules (with $\psi \in N^+$ and $\psi \in T^*$, respectively), and S never occurs in the right-hand side of a rule. For $\phi \in N^+$ and $\psi \in N^* \cup T^*$, ϕ

derives ψ in one step (denoted $\phi \Rightarrow \psi$) if and only if $\phi = A_1 A_2 \cdots A_n$ and $\psi = \psi_1 \psi_2 \cdots \psi_n$ for some rules $(A_i \to \psi_i) \in R$. The relations $\Rightarrow^n$ and $\Rightarrow^*$ are defined in the usual way, denoting derivations of n steps and arbitrary derivations, respectively. For $A \in N$, $L(G, A) = \{w \in T^* \mid A \Rightarrow^* w\}$, and $L(G) = L(G, S)$. EOL denotes the class of languages generated by EOL systems, i.e., $\mathrm{EOL} = \{L(G) \mid G$ is an EOL system$\}$. Note that one can derive strings using G in a different way. In particular one can use G as a context-free grammar by redefining $\Rightarrow$ appropriately. We denote by $L_{\mathrm{CF}}(G)$ the language generated by G in such a way. Hence, depending on the way that G is used, we will refer to G both as an EOL system and as a context-free grammar.

Thus one derivation step of the EOL system G consists of the parallel application of several rules of the context-free grammar G. Hence the set of derivation trees of the EOL system consists of all those derivation trees of the context-free grammar such that all paths from the root to a leaf (not labeled λ) have the same length. In particular $L(G) \subseteq L_{\mathrm{CF}}(G)$. It is easily seen that if R contains a rule $A \to A$ for every $A \in N$, then $L(G) = L_{\mathrm{CF}}(G)$, and so $\mathrm{CF} \subseteq \mathrm{EOL}$.

Although our definition of an EOL system is not standard, it corresponds in a straightforward way to the notion of a synchronized EOL system, see Theorem II.1.7 of [9]. This way of defining an EOL system was already used in [1], where it is called an FMOL system.

A well-known basic technique to deal with EOL systems is that of slicing or speeding up an EOL system [8, 9]. We need this technique to ensure that, for every nonterminal A, A generates a terminal string for every number of steps; moreover, we want that, in the sliced system, A does not generate more than in the original system. See also the notion of a "neatly synchronized system" in Lemma II.4.1 of [9].

LEMMA. *For every EOL system $G = (N, T, R, S)$ there is an EOL system $G^1 = (N^1, T, R^1, S)$ with $N^1 \subseteq N$, such that $L(G^1) = L(G)$ and for every $A \in N^1 - \{S\}$,*

(i) *for each $n \geq 1$ there exists $w \in T^*$ such that $A \Rightarrow^n w$ in G^1,*

(ii) $L(G^1, A) \subseteq L(G, A)$.

Proof. The proof goes along well-known lines. For $A \in N$, the existential spectrum of A in G is defined by $\mathrm{espec}(A) = \{n \geq 1 \mid A \Rightarrow^n w$ for some $w \in T\}$. By Theorem II.1.6 of [9], $\mathrm{espec}(A)$ is an ultimately periodic set of integers. Let k be the "uniform period" of G (see the proof of Theorem II.2.2 of [9]), i.e., k is any number such that, for every A, k is a multiple of the period of $\mathrm{espec}(A)$ and k is larger than the initial nonperiodic part of $\mathrm{espec}(A)$. Define G^1 to be $\mathrm{speed}_k(G)$, see the end of Section II of [9], i.e., $G^1 = (N, T, R^1, S)$, where $R^1 = \{S \to \psi \mid S \Rightarrow^i \psi$ for some i, $1 \leq i \leq k\} \cup$

$\{A \to \psi \mid A \Rightarrow^k \psi,\ A \neq S\}$. Clearly $L(G^1) = L(G)$, and every nonterminal generates in G^1 a subset of what it generates in G (property (ii)). Moreover, it should be clear from the properties of k that, for $A \neq S$ and all $n, m \geqslant 1$, $nk \in \text{espec}(A)$ if and only if $mk \in \text{espec}(A)$, and hence $n \in \text{espec}^1(A)$ if and only if $m \in \text{espec}^1(A)$, where $\text{espec}^1(A)$ is the existential spectrum of A in G^1.

Consequently either $\text{espec}^1(A) = \varnothing$ (and so $L(G^1, A) = \varnothing$) or $\text{espec}^1(A)$ contains all $n \geqslant 1$. After removing all nonterminals with empty existential spectrum from G^1, only nonterminals remain (in some N^1) which generate terminal strings for every number of steps (property (i)). ∎

3. The Translational Theorem

We now prove our main result.

THEOREM 1. *Let K be a language over alphabet T and $a \notin T$. If* $\text{sh}(K, a^*) \in EOL$, *then* $K \in CF$.

Proof. Let $G = (N, T \cup \{a\}, R, S)$ be an EOL system such that $L(G) = \text{sh}(K, a^*)$. We have to show that $\text{pres}_T(L(G)) \in \text{CF}$. The proof consists of the successive construction of EOL systems G_1, G_2 and G_3 such that $\text{pres}_T(L(G_i)) = \text{pres}_T(L(G)) = K$. Moreover, for G_3 we will show that $\text{pres}_T(L(G_3)) = \text{pres}_T(L_{\text{CF}}(G_3))$, which shows that $K \in \text{CF}$ (it is obtained from the context-free language $L_{\text{CF}}(G_3)$ by the homomorphism pres_T).

First we define $G_1 = (N_1, T \cup \{a\}, R_1, S)$ with $N_1 = \{S\} \cup \{A_e \mid A \in N - \{S\}\} \cup \{A_t \mid A \in N - \{S\},\ t \in T\} \cup \{A_m \mid A \in N - \{S\}\}$. Thus each nonterminal A (except S) is replaced by new nonterminals A_e, A_m, and A_t, for every $t \in T$, with the following intended interpretation:

$$L(G_1, A_e) = L(G, A) \cap a^*,$$

$$L(G_1, A_t) = L(G, A) \cap a^*ta^* \text{ for every } t \in T,$$

$$L(G_1, A_m) = L(G, A) \cap \Sigma^*T\Sigma^*T\Sigma^*, \text{ where } \Sigma = T \cup \{a\}.$$

Informally, A_e generates no element of T, A_t generates one element of T (viz. t), and A_m generates two or more elements of T. The construction of R_1 is straight-forward and exactly the same as for context-free grammars (where this is an example of the wellknown technique of "adding information to the nonterminals"). A terminal rule $A \to w$ (with $A \neq S$) of R is replaced by a terminal rule $A_p \to w$ of R_1, where $p = e$, t or m if $w \in a^*$, $w \in a^*ta^*$ or $w \in \Sigma^*T\Sigma^*T\Sigma^*$, respectively. A nonterminal rule $A \to A^1A^2 \cdots A^k$ (with $A \neq S$) of R is replaced by all nonterminal rules $A_p \to A^1_{p_1}A^2_{p_2} \cdots A^k_{p_k}$ of R_1 such that

$p = m$ if at least one p_i is m or at least two p_i are in T,

$p = t$ if, for some i, $p_i = t$ and $p_j = e$ for all $j \neq i$,

$p = e$ if $p_i = e$ for all i.

The terminal rules for S in R_1 are the same as those in R, and the nonterminal rules for S in R_1 are obtained from those in R by adding all possible subscripts from the set $\{m, e\} \cup T$ to the right-hand side nonterminals. This ends the construction of G_1. It is left to the reader to see that $L(G_1) = L(G)$ and that the intended equalities are satisfied.

Next, let $G_2 = (N_2, T \cup \{a\}, R_2, S)$ with $N_2 \subseteq N_1$ be the EOL system G_1^1 resulting from G_1 as in the Lemma of Section 2. Note that $L(G_2) = L(G_1) = L(G) = \mathrm{sh}(K, a^*)$. Note also that, by property (ii) of the Lemma, we still know that for all nonterminals of G_2 (except S)

$$
\begin{aligned}
&L(G_2, A_e) \subseteq a^*, \\
&L(G_2, A_t) \subseteq a^*ta^* \qquad \text{for every } t \in T, \qquad\qquad (*) \\
&L(G_2, A_m) \subseteq \Sigma^*T\Sigma^*T\Sigma^*.
\end{aligned}
$$

Finally we construct $G_3 = (N_2, T \cup \{a\}, R_3, S)$ simply by removing from R_2 all terminal rules of the form $A_m \to w$ ($A_m \in N_2$, $w \in (T \cup \{a\})^*$). In general this changes the generated language, so we can say only that $L(G_3) \subseteq L(G_2)$. However, $\mathrm{pres}_T(L(G_3)) = \mathrm{pres}_T(L(G_2))$. To see this, note that for every string $t_1 t_2 \cdots t_k \in K$ ($t_i \in T$) the string $t_1 a^n t_2 a^n \cdots a^n t_k$ is in $\mathrm{sh}(K, a^*)$, where n is an integer larger than the length of the right-hand side of any terminal rule of G_2. Clearly this string is generated by G_3: in the derivation of it in G_2 no terminal rule $A_m \to w$ can be used, because w contains at least two elements of T.

Obviously the inclusions $(*)$ also hold for G_3. Note that in G_3 all nonterminals A_e and A_t still satisfy property (i) of the Lemma, because, by $(*)$, in derivations starting from A_e or A_t no nonterminal with subscript m can be used.

It remains to show that $\mathrm{pres}_T(L(G_3)) = \mathrm{pres}_T(L_{\mathrm{CF}}(G_3))$. Since $L(G_3) \subseteq L_{\mathrm{CF}}(G_3)$ it suffices to prove that $\mathrm{pres}_T(L_{\mathrm{CF}}(G_3)) \subseteq \mathrm{pres}_T(L(G_3))$. Consider a derivation tree d of the context-free grammar G_3. We want to show that there is a derivation tree d^1 of the EOL system G_3 such that d^1 contains the same elements of T at its leaves as d (in the same order). By the construction of G_3, all father nodes of (terminal) leaves of d are labelled by nonterminals A_e or A_t (or S, but in that case d is an EOL derivation tree already). Note that the terminal strings generated by these A_e and A_t in d are in $L(G, A_e)$ and $L(G, A_t)$, respectively (because they are one-step derivations). Moreover, since they EOL-generate terminal strings in every number of steps, we can replace their subtrees (of depth 1) by other subtrees such that the resulting tree d^1 is a derivation tree of EOL system G_3. To be precise, if d has depth k and, e.g., A_e is at depth p in d, then A_e should have a new subtree of depth $k-p$ (corresponding to some EOL derivation of $k-p$ steps); thus d^1 will also have depth k. Since, by (*), $L(G_3, A_e) \subseteq a^*$ and $L(G_3, A_t) \subseteq a^*ta^*$, d^1 contains the same elements of T as d. This concludes the proof of the theorem.

For an example of the last argument, see Fig. 1: d is a context-free derivation tree such that $\text{pres}_T(\text{yield}(d)) = t_1 t_2$. The nonterminal nodes of d and the nodes labeled by elements of T are indicated by dots, the terminal rules by small triangles; in d^1 the new subtrees are indicated by large triangles. The depth of d is 7 and, e.g., the new subtree of B_{t_1} has depth 5. ∎

Note that the theorem is optimal in the sense that CF cannot be replaced by any smaller subclass of EOL. This is because CF is closed under the operation $\text{sh}(-, a^*)$.

Finally we state some consequences of Theorem 1. First of all, there is no full AFL between CF and EOL, and even, CF is the largest.

COROLLARY 1. *CF is the largest full AFL contained in EOL.*

Proof. Let $\mathscr{L}$ be a full AFL, $\mathscr{L} \subseteq$ EOL. Let $K \in \mathscr{L}$. Since $\mathscr{L}$ is a full AFL, also $\text{sh}(K, a^*) \in \mathscr{L}$. Hence $\text{sh}(K, a^*) \in$ EOL and, by Theorem 1, $K \in$ CF. Thus $\mathscr{L} \subseteq$ CF. ∎

In fact the proof of this corollary shows that CF is even the largest class in EOL closed under inverse homomorphisms. The same is true for closure under regular substitution.

Theorem 1 also indicates in some sense the precise amount of nonclosure of EOL under substitution. For classes $\mathscr{L}_1$ and $\mathscr{L}_2$ of languages, let $\mathscr{L}_1 \to \mathscr{L}_2$ denote the class of languages obtained by substituting languages from $\mathscr{L}_1$ into a language from $\mathscr{L}_2$. Let FIN denote the class of finite languages. Since EOL is closed under finite substitution, FIN $\to$ EOL $\subseteq$ EOL. It is also easy

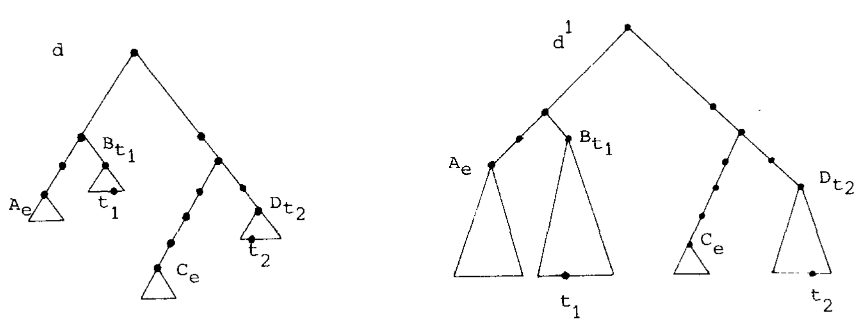

FIG. 1. Transforming a context-free derivation tree d into an EOL derivation tree d^1.

to prove that EOL $\to$ CF $\subseteq$ EOL, i.e., EOL is closed under substitution into context-free languages. By Theorem 1, these results are optimal, i.e., FIN and CF are the largest allowable classes in these two inclusions. In fact, "marked substitution" of an infinite language into a non-context-free language leads out of EOL. For languages K_1 and K_2 over disjoint alphabets, the *marked substitution* $\tau(K_1, K_2)$ of K_1 into K_2 is $\phi(K_2)$, where ϕ is the substitution such that $\phi(t) = tK_1$ for all t in the alphabet of K_2. The previous statement can now be formulated in the following translational theorem (cf. [4], where marked substitution is considered for arbitrary AFLs).

COROLLARY 2. *Let K_1 and K_2 be languages over disjoint alphabets. If $\tau(K_1, K_2) \in EOL$, then $K_1 \in FIN$ or $K_2 \in CF$.*

Proof. Let $\tau(K_1, K_2) \in$ EOL and assume that K_1 is infinite. Using the known closure properties of EOL it is easy to see that this implies that $\mathrm{sh}(K_2, a^*) \in$ EOL. In fact, to obtain $\mathrm{sh}(K_2, a^*)$ from $\tau(K_1, K_2)$ it suffices to change every symbol of K_1 into a or λ, and to add a^* in front. Hence, by Theorem 1, $K_2 \in$ CF. ∎

4. A GENERALIZATION

In this section we reformulate Theorem 1 and its proof in a somewhat more general way and demonstrate that in the new form it yields the main result of [2] as a corollary. To do this we need the following terminology.

Let K_1 be a language over alphabet Σ and let $T \subseteq \Sigma$. For $n \geqslant 1$, $R_n(K_1, T)$ denotes the language over Σ defined by $R_n(K_1, T) = \{w_0 t_1 w_1 t_2 w_2 \cdots w_{m-1} t_m w_m \in K_1 \mid m \geqslant 0, t_j \in T, w_j \in (\Sigma - T)^*$, and for all i, $1 \leqslant i \leqslant m-1$, $|w_i| \geqslant n\}$. Furthermore, $L_n(K_1, T) = \mathrm{pres}_T(R_n(K_1, T))$. Thus, $L_n(K_1, T) = \{t_1 t_2 \cdots t_m \in T^* \mid$ there exist $w_0, w_1 \cdots w_m \in (\Sigma - T)^*$ such that $w_0 t_1 w_1 \cdots t_m w_m \in K_1$ and for all i, $1 \leqslant i \leqslant m-1$, $|w_i| \geqslant n\}$. In other words, $L_n(K_1, T)$ consists of all strings over T which occur in strings of K_1 for which the distance between symbols of T is at least n.

THEOREM 2. *Let K_1 be an EOL language over alphabet Σ, and $T \subseteq \Sigma$. Then there exist an integer $n \geqslant 1$ and a context-free language L over T such that $L_n(K_1, T) \subseteq L \subseteq \mathrm{pres}_T(K_1)$.*

Proof. The proof is entirely analogous to the proof of Theorem 1, treating each element of $\Sigma - T$ as a, K_1 as $\mathrm{sh}(K, a^*)$, and $\mathrm{pres}_T(K_1)$ as K. The construction of G_1, G_2 and G_3 is the same. Clearly $L(G_2) = L(G_1) = L(G) = K_1$, as before. It is not true any more that $\mathrm{pres}_T(L(G_3)) = \mathrm{pres}_T(L(G_2))$. However, $R_n(L(G_2), T) \subseteq L(G_3) \subseteq L(G_2)$, where n is the maximal length of the right-hand sides of the terminal rules of G_2, by a similar argument. Hence, applying pres_T to these inclusions, $L_n(K_1, T) \subseteq \text{pres}_T(L(G_3)) \subseteq \text{pres}_T(K_1)$. Since $\text{pres}_T(L(G_3)) = \text{pres}_T(L_{\text{CF}}(G_3))$ as before, $L = \text{pres}_T(L(G_3))$ is a context-free language. ∎

Note that, if in the above theorem $K_1 = \text{sh}(K, a^*)$, then, for any n, $L_n(K_1, T) = \text{pres}_T(K_1)$. Hence $K = \text{pres}_T(K_1) = L$ and so K is context-free. Thus Theorem 1 is an immediate consequence of Theorem 2.

We now show that Theorem 2 is a (strict) generalization of the result of [2]. Let, again, K_1 be a language over Σ, and $T \subseteq \Sigma$. We say that T is *clustered* in K_1 if $L_n(K_1, T)$ is finite for some $n \geqslant 1$. It is easy to see that this definition is equivalent to the one in [2].

COROLLARY 3. *Let K_1 be an EOL language over alphabet Σ, and $T \subseteq \Sigma$. If* $\text{pres}_T(K_1)$ *does not contain an infinite context-free language, then T is clustered in K_1.*

Proof. By Theorem 2, since $\text{pres}_T(K_1)$ does not contain an infinite context-free language, $L_n(K_1, T)$ has to be finite. ∎

We say that a set I of integers is "of exponential nature" if for all k there exists m_k such that for all $x, y \in I$ larger than m_k, if $x \neq y$ then $|x - y| \geqslant k$. Following [2] we say that T is *numerically dispersed* in K_1 if the length-set of $\text{pres}_T(K_1)$, i.e., the set $\{i \mid i = |w| \text{ for some } w \in \text{pres}_T(K_1)\}$, is of exponential nature.

COROLLARY 4 [2]. *Let K_1 be an EOL language over Σ, and $T \subseteq \Sigma$. If T is numerically dispersed in K_1, then T is clustered in K_1.*

Proof. Suppose that T is not clustered in K_1. Then, by Corollary 3, $\text{pres}_T(K_1)$ contains an infinite context-free language. Thus, by the pumping lemma for context-free languages, its length-set contains an arithmetic progression of integers. Hence T is not numerically dispersed in K_1. ∎

It should be clear that Theorem 2 (and Corollary 3) is stronger than Corollary 4. For example, using Corollary 4 it is not possible to show that $\text{sh}(K, a^*) \notin \text{EOL}$, where $K = \{b^{2^n} \mid n \geqslant 0\} \cup \{b^{2^n+1} \mid n \geqslant 0\}$, and certainly not that $\text{sh}(K^1, a^*) \notin \text{EOL}$, where $K^1 = \{b^n c^n d^n \mid n \geqslant 1\}$. Moreover, we feel that our proof of Theorem 2 is much shorter and more intuitive than the proof of Corollary 4 in [2].

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